

It must be emphasized that the preceding example represents an extremely inefficient application of the method. Nevertheless, the accompanying progressive results should enable the reader to obtain a clearer understanding of the approach.

Note that after submitting the original manuscript, the authors received the paper by D. Kavlie and G. H. Powell.⁸ The reader may be referred to this paper for a most thorough comparison of current methods. The new method presented in Eqs. (46-51) of that paper is most elegant, and is the fastest direct method presented there. However, as in Ref. 5, it contains an unsymmetrical transformation plus back-substitution, so that the operation count (neglecting load dependent terms) in Eq. (51), even when reduced by considering an average distribution of modifications, to

$$n \approx 3NBn_c/2 + Nn_c^2/4 \quad (32)$$

implies more than double the operations given in the present Eq. (21).

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Allen and Vincenti Blockage Corrections in a Wind Tunnel

C. DALTON*

University of Houston, Houston, Texas

Introduction

IN conducting wind-tunnel tests it is quite often necessary to use cylinders (or bodies) a little larger than desirable in order to attain the highest possible Reynolds number. Use of the large cylinders gives rise to wall-interference effects which, of course, influence whatever measurement is desired. There are several techniques which might be followed so that the wall-interference effects might be eliminated.

One of the most popular procedures for obtaining corrected drag forces for a single cylinder from wind-tunnel data is due to Allen and Vincenti.¹ For example, the Allen and Vincenti (A & V) procedure has been used by Bishop and Hassan,²

Roshko,³ and Delany and Sorensen⁴ and Achenbach⁷ to correct the measured drag coefficient on circular cylinders in wind-tunnel tests. In fact, Roshko states that the A & V procedure is the best correction method available and is believed to be fairly accurate at supercritical Reynolds numbers where C_d is nearly constant.

A recent investigation by the author led to the discovery that the A & V method is noticeably in error when used with relatively large-diameter circular cylinders in a wind tunnel.

Allen and Vincenti Analysis

When a cylinder is placed in a wind tunnel, the flowfield is influenced to the extent that the unbounded plane-flow situation is no longer modeled exactly. Allen and Vincenti¹ performed an analysis which yielded the following equation [Eq. (67) in Ref. 1] to represent the actual drag coefficient,

$$C_d = C_d' \left\{ 1 - \frac{(2 - M^2)}{(1 - M^2)^{3/2}} \Lambda \sigma - \frac{(1 + 0.4M^2)}{(1 - M^2)^{3/2}} \Lambda \sigma - \tau C_d' \frac{(2 - M^2)(1 + 0.4M^2)}{1 - M^2} \right\} \quad (1)$$

in which Λ is a shape factor and σ and τ are geometric factors all tabulated in Ref. 1, C_d' is the measured drag coefficient, and M is the apparent upstream Mach number. To explain the terms in Eq. (1), I quote from Allen and Vincenti¹ that "... of the two correction terms involving $\Lambda \sigma$ in this equation, the first appears as a result of the change in dynamic pressure occasioned by the interference between the walls and the airfoil thickness; the second represents the effect of the pressure gradient induced by the interference between the walls and the wake. The correction term containing $\tau C_d'$ appears as a result of the change in dynamic pressure caused by the wall-wake interference." Very good agreement with drag data was obtained by Allen and Vincenti for cylindrical airfoils over a range of Mach numbers and airfoil sizes.

For the case of a circular cylinder in a wind tunnel with a negligible Mach number ($M \lesssim 0.2$), Eq. (1) reduces to

$$C_d = C_d' \{ 1 - 2.472(d/h)^2 - 0.5C_d'(d/h) \} \quad (2)$$

in which d is the cylinder diameter and h is the wind-tunnel width for a vertical cylinder. Equation (2) was also listed by Roshko³ in which the coefficient 2.472 was replaced by 2.5 which certainly is acceptable.

The data of Fage⁵ were used by Allen and Vincenti to obtain corrected drag coefficients in Eq. (1) for flow around a circular cylinder and an apparent computational error was made. I have calculated from Eq. (2) the corrected drag coefficients and in Fig. 1 a plot is shown of the uncorrected Fage data and the drag coefficient values as corrected using Eq. (2). Superimposed on the figure are the corrected values as contained

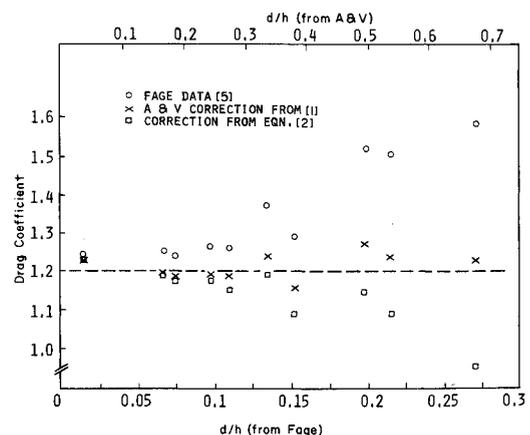


Fig. 1 Corrected and uncorrected drag coefficients as a function of blockage.

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* Associate Professor of Mechanical Engineering.

in the A & V report. It is clear from the figure that there is a distinct difference between the corrected values taken directly from Fig. 13 in Ref. 1 and the corrected values computed from Eqs. (1) or (2) [also Eq. (67) in Ref. 1] herein. Note that the difference between the two sets of corrected values increases as the (d/h) ratio increases. These two sets of numbers should be the same since the values taken from the A & V report were determined from Eq. (1) in simplified form [which is Eq. (2)]. I would also like to point out that the abscissa on Fig. 13 of Ref. 1 is incorrect for circular cylinders. The lower abscissa on our Fig. 1 is taken from Fage's data; the upper abscissa on our Fig. 1 is that given by Allen and Vincenti. The lower abscissa on this figure is the correct one to use.

Based on Fage's experimental data, the plotted values of Allen and Vincenti for the freefield drag coefficient C_d are very close to the expected value of 1.2. My calculations with Eq. (2) [or, equivalently, Eq. (67) in Ref. 1] and Fage's data have shown that the freefield drag-coefficient values of C_d are increasingly less than 1.2 as the relative spacing ratio (d/h) increases. Note that these deviations from the expected value of 1.2 become significant for (d/h) spacings greater than about 0.1.

I feel that the reason for the deviation of the A & V calculations from the results obtained from Eq. (2) herein has been found. In what was intended to be a simple check of computational procedure, I recalculated the A & V results, obtaining their values for the corrected coefficients. However, after completion of these calculations, it was noticed that, in using Eq. (2), a mistake was made on one of the coefficients. Instead of using 2.472, I had used 0.2472 and had obtained the A & V values. Upon repeating the calculations with the proper tabulated value of 2.472, I found the corrected values to be those shown in Fig. 1 and labeled "Correction from Eq. (2)." Therefore, it seems that the A & V method produced good agreement with the data because of a simple decimal oversight in the calculational procedure.

Summary

These observations indicate that the A & V blockage corrections for drag coefficients on a circular cylinder in a wind tunnel should not be used for spacing ratios greater than 0.1. It would appear that the potential-flow model as posed by Allen and Vincenti does not accurately represent the real-flow situation for a circular cylinder when the spacing ratio becomes too large. However, I did find that the A & V method gives good agreement with the data for all available spacing ratios when the following equation is used instead of Eq. (2):

$$C_d = C_d' \{ 1 - \frac{1}{4}(d/h)^2 - (C_d'/2)(d/h) \} \quad (3)$$

I suggest that Eq. (3) be used as a replacement for the A & V method for computation of blockage-corrected drag coefficients. I make this suggestion with no intent to cast doubt on the theoretical analysis set forth by Allen and Vincenti, but simply to have available a calculational procedure for blockage corrections which produces results in agreement with free-field values.

As an alternate method for circular-cylinder blockage corrections with large spacing ratios, I suggest that the method of Fage³ as outlined in Durand⁶ be used.

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Influence of the Injection Conditions on the Ignition of Methane and Hydrogen in a Hot Mach 2 Air Stream

K. BIER,* G. KAPPLER,† AND H. WILHELMI‡
University of Karlsruhe, Karlsruhe, Germany

IN a preceding investigation the initiation and propagation of combustion was studied for transverse fuel injection into supersonic air streams heated by a 350 kW plasma burner.¹ Gaseous hydrogen and methane were injected through a cylindrical nozzle of 1.56 mm ϕ at an angle of 90° into free, parallel air streams with Mach numbers between 2 and 3 and static temperatures ranging from $\approx 600^\circ$ to $\approx 2000^\circ\text{C}$. In the following, further experiments concerning the influence of the injection angle and of an adjacent wall on the ignition temperature are described.

The experiments were conducted with the same set-up as used in the previous investigation. Figure 1 shows the geometrical arrangement for the injection of the fuel gas through an inclined cylindrical nozzle of $d_s = 1.5$ mm diam and through a vertical cylindrical hole of the same diameter in a flat water-cooled copper plate, adjusted tangentially to the air flow. In the latter case, ambient air could be introduced through a slit of $s = 0.4$ mm width into the wake behind the fuel gas jet. As in the previous investigation, the "ignition temperature" t_i , that is the minimum static temperature of the undisturbed air stream causing ignition, was determined as a function of the fuel pressure ratio p_{os}/p_k (p_{os} = stagnation pressure of the fuel, p_k = static pressure in the undisturbed air stream, equal to atmospheric pressure). The stagnation temperature of the fuel was always 20°C .

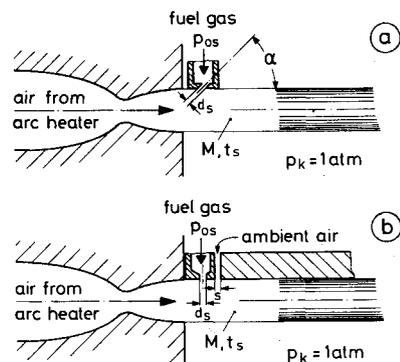


Fig. 1 a) Schematic of the fuel injection through an inclined nozzle and b) a boring in a plate.

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 * Professor of Thermodynamics.
 † Research Assistant.
 ‡ Oberingenieur, Thermodynamik Institut.